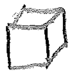
P302 Photonics Fall 2010
Solutions to HW # 12

(13.1)  $l = 10\text{ cm} = 0.1\text{ m}$ $\quad T = 400°C = 673\text{ K}$

$A = l^2 = 10^{-2}\text{ m}^2$ $\quad \varepsilon = 0.97$

$$P = \varepsilon\sigma A T^4 = (0.97)(5.67\times10^{-8}\tfrac{W}{m^2K^4})(10^{-2}m^2)(673\text{ K})^4$$

$$\underline{P = 113\text{ W}}$$

(13.7) From eq 13.12 $\quad I = a\nu^3(e^{b\nu}-1)^{-1}$

where $a = 2\pi h/c^2$

$b = h/k_BT$ $\quad$ and $T = 40\times10^3\text{ K}$

To find ν_{max} when I is maximum, can do in several ways:

a) $\frac{dI}{d\nu} \overset{set}{=} 0$, solve for ν. You end up with the equation $e^{b\nu}(3-\nu b) = 3$
which is not easily solved. One way is by plotting.

b) Go online to a black body calculator, and find $\lambda_{max} = 0.07244\,\mu m \Rightarrow \underline{\nu_{max} = \frac{c}{\lambda_{max}} = 4.14\times10^{15}\text{ Hz}}$

c) Wein's Law: $\lambda_{max}T = \text{constant} = 2.8977\times10^{-3}\text{ m}\cdot\text{K}$

So $\lambda_{max} = 0.07244\,\mu m$

or, again, $\underline{\nu_{max} = 4.14\times10^{15}\text{ Hz}}$

13.11 (a) $\mathcal{E} = h\nu = hc/\lambda$

But $1 eV$ = energy gained by an electron accelerated by one volt

$= (1 q_e)(1V)$

$= (1.602 \times 10^{-19} C)(1V) = 1.602 \times 10^{-19} J$

So $\mathcal{E} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} J.s)(3.00 \times 10^{8} \frac{m}{s})}{\lambda} \left(\frac{1eV}{1.602 \times 10^{-19} J} \right)$

$$\mathcal{E} = \frac{1.24 \times 10^{-6} \, eV.m}{\lambda} = \frac{1.24 \times 10^{3} \, eV.nm}{\lambda}$$

(b) for $\lambda = 600 nm$

$\mathcal{E} = 2.07 \, eV$

13.17 From problem 13.16

$$\alpha \overset{=}{_D} \frac{(dN_j/dt)_{st}}{(dN_j/dt)_{sp}} = \left(e^{h\nu/k_BT} - 1\right)^{-1}$$

for $h\nu = 2.0 eV$, $T = 300K$

$h\nu/k_BT = 77.3$ (dimensionless)

so $\alpha = \frac{1}{e^{77.3} - 1}$

or $\alpha \cong e^{-77.3} = 2.69 \times 10^{-34}$

(13.17) Cont'd

Since the stimulated emission rate is much smaller than the spontaneous emission rate, the excited states will very quickly be de-populated by spontaneous emission. No population inversion is possible.

(13.24) $\lambda = c/\nu \Rightarrow \Delta\lambda = \frac{c}{\nu^2}\Delta\nu = \left(\frac{c}{\nu}\right)^2 \frac{\Delta\nu}{c}$

or $\Delta\lambda = \lambda^2 \Delta\nu / c$

with $\lambda = 694.3\ nm$

$\Delta\nu = 50\ MHz$

$\Rightarrow \underline{\Delta\lambda = 8.03 \times 10^{-14}\ m = 8.03 \times 10^{-5}\ nm}$